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Mathematical Reasoning and Common-sense in Word Problem-solving

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ABSTRACT The purpose of this paper is to explore the common tendency of learners to relegate reality from their solution processes when they engage in real wor(l)d problem-solving. The study was conducted in township mathematics classrooms contexts with learners drawn from different socio-cultural backgrounds. The data collection strategies for the purpose of this study included a test and focus groups discussions. The results of this study demonstrate the importance of connecting formal classroom mathematics activities to learners' out-of-school real world knowledge and experiences during problem solving. Moreover, the study illustrates the ways in which the learners include and use cultural knowledge to arrive at and justify solutions to mathematical problems.

INTRODUCTION

There has been much recent global interest in problem-solving in relation to socio-cultural perspectives in mathematics education (Greer 1997; Inuoe 2009; Säljö et al. 2009; Verschaffel et al. 2009). Such interest, which reflects an acknowledgement of the complex nature of classroom environments and cultural aspects of learning and teaching mathematics, is highly significant for mathematics education in South Africa due to our predominantly multilingual settings (see Setati 1998; Adler 2001; Barwell and Setati 2005). In fact, many researchers (for example, Verschaffel et al. 1994; Greer 1997; Yoshida et al. 1997; Xin and Zhang 2009) have studied the tendency of learners to provide unrealistic solutions to real wor(l)d problems without considering common aspects of reality. An increasing number of researchers (for example, Cooper and Harries 2005; Xin and Zhang 2009) have consistently suggested that current school instruction for arithmetic word problems is likely to develop in students the tendency to relegate real-world knowledge and realistic considerations from their solution processes, and actually promotes suspension of sense-making (Schoenfeld 1991).

There is an overwhelmingly poor performance in mathematics word problem-solving by South African ninth grade learners in mathematics classrooms, consistent with findings of studies conducted elsewhere in the world (see for example Verschaffel et al. 2009). This poor performance in word problem-solving appears to be as a result of the way mathematics word problems are addressed in mathematics classrooms. Learners' attempts to solve word problems reflect the mechanical methods of (or approaches to) solving word problems as promoted by the school mathematics textbooks used by mathematics teachers during teaching and assessments. In consequence, learners have a tendency to exclude reality in their solution processes, generating conclusions that are mathematically correct but situationally inappropriate (or inaccurate) since they do not make sense in real world (Sepeng and Webb 2012; Sepeng 2013).

In this paper therefore it is argued that it is important for all practitioners (mathematics teachers, examiners, curriculum designers, mathematics textbook authors, and learners) to be aware of the roles played by two distinct discourses in mathematics word problem-solving, namely: (1) classroom mathematics in which word problems are solved mechanically, and (2) reality based reasoning, where learners'out-of-classroom knowledge (or everyday knowledge and experience), is brought to bear on the problem-solving task. This, in turn raises the further issue of the socio-cultural (and/or linguistic) differences found in multilingual mathematics classrooms.

It is important for all practitioners to understand that the above-mentioned two discourses extend across the whole domain of word problem-solving because they affect teaching, learning, and assessment in general. For example, authors and examiners should take cognisance of the type of word problems used as examples in textbooks and for assessments in examination question papers. In both cases learners' interpretations of the situation embedded within word problems may differ based on their social knowledge backgrounds, and as a result, this may advantage or disadvantage certain groups of learners.

This paper begins with a discussion of the current debates on connections between classroom mathematics (or classroom activities) and second language learners' everyday experiences, and the value of connecting what is taught and learned in mathematics classrooms with the out-of-classroom learning. In particular, much of the work in this paper relies both theoretically and methodologically on notions of classroom mathematics discourse and mathematical modelling. The main argument in this paper is that implicit beliefs and rules relating specifically to learners' mathematical activities hinder learners from using realistic knowledge in their solutions. The purpose of this paper is to illustrate how learners' justifications of their unrealistic solutions can inform practitioners, such as mathematics teachers, curriculum designers, and policy makers, of the various ways in which they clarify and make sense of the problem situation, as well as the nature of problem solving activity.

Connections between Classroom Activities and Real-Life Experiences

The connection between classroom mathematics and learners' everyday experiences is a complex issue because the two contexts differ significantly. Lave (1992) suggests that word problem-solving describes stylised representations of hypothetical experiences separated from the students' experiences. In word problem-solving, students' minds could be torn between two types of knowledge system that the word problem activates – one developed in the traditional mathematics classroom and the other developed through real-world experiences (Inoue 2005). Inoue claims that in traditional schooling, students are not asked to examine different sets of assumptions for solving mathematical word problems.

For many children in elementary school, emphasis has been put on syntax and arithmetic rules rather than treating the problem statement as a description of some real-world situation to be modelled mathematically (Xin 2009). For example, studies (Liu and Chen 2003) conducted on 148 Chinese students from 4^{th} and 6^{th} grade, reported that only one fourth (26%) of the students' solutions of problems were from a realistic point of view (attending to realistic considerations). Almost half (48%) of the responses revealed a strong tendency to exclude real world knowledge, and in the rest of the cases, no answer was given. According to Inoue (2009), the unrealistic solutions may not simply stem from mindless or procedural problem solving, but could originate in students' diverse effort to make sense of the problem situation and the nature of the problem solving activity in sociocultural contexts. In fact, Verschaffel et al. (2000) have suggested that many students whose problem solving did not seem to reflect familiar aspects of reality are known to defend their answers when their attention is drawn to the issue. Inoue (2005) argues that looking into students' justifications of their seemingly unrealistic answers can inform us of the various ways in which students interpret and make sense of the problem situation as well as the nature of problem solving activity.

Word problem-solving in school contexts serves as a game under tacitly agreed upon rules of interpretation (Greer 1997). According to Gatto (1992), these agreed rules are internalised in the students' minds through the socio-mathematical norm, or hidden curriculum of traditional schooling that could influence many aspects of the intellectual activities in schools. Inoue (2009) suggests that instead of dismissing students' computational answers, examining different sets of assumptions for solving word problems can provide rich opportunities for students to learn how to use their mathematical knowledge beyond school-based problem solving. Inoue points out that this could help the students conceptualise word problem solving in terms of meaningful assumptions and conditions for modelling reality, rather than the assumptions imposed by textbooks, teachers, or authority figures.

Mathematical Reasoning through Argumentation and Discussion

Argumentation in classroom contexts encompasses a process where learners make a claim, provide suitable evidence to justify it, and defend the claim logically until a meaningful decision has been reached (Webb et al. 2008). Argumentation is viewed as a technique that may be employed to enhance mathematical reasoning and maximise learner participation in mathematics classrooms through discussion (Sepeng and Webb 2012). The use of discussion as a tool to increase reasoning has gained emphasis in classrooms worldwide, consistent with earlier reports (Yore et al. 2003). Discussion, however, requires scaffolding and structure in order to support learning (Norris and Phillips 2003).

Wood et al. (2006) found variation in students' ways of seeing different contexts and reasoning in problem solving, and these were assigned in the first place to the particular differences established in classrooms early in the year pertaining to *when* and *how to contribute* to mathematical discussions and *what to do as a listener*.

Theoretical Perspective

This study is framed by a socio-cultural perspective (Cooper 1998). The socio-cultural perspective proposes that collective and individual processes are directly related, and students' unrealistic responses to real world problems reflects the students' socio-cultural relationship to school mathematics and their willingness to employ the approaches emphasised in school. From a socio-cultural perspective, modelling implies engaging in inter-semiotic work. In other words, one has to decide about the appropriate and useful manners of coordinating linguistic categories and mathematical expressions and operations in order to come to a solution problem (Säljö et al. 2009). In intersemiotic meaning-making, the truth value of statements and arguments is established on the basis of analytical considerations of how a particular usage of concepts fits into the universe of meaning that is mathematical discourse.

METHODOLOGY

Participants

Participants in this study were 107 ninth grade learners from township schools in the Eastern Cape of South Africa. These learners were drawn from different social-cultural backgrounds. The schools attract learners from poor to low-income households, and families receiving social grants from the government. Data gathering included tests and focus groups discussions, supported by field notes. Learners' responses during group and individual problem-solving of the problem-solving task were videotaped and later transcribed in full. These methods of data gathering are appropriate for research designed within a socio-cultural perspective because they allow for the opportunity to examine classroom discourse and to make sense of how learners reason and communicate mathematically. In other words, the study followed a mixed-methods design with qualitative results informing quantitative data. This design was selected because the researcher wanted to understand not only how learners solved the tasks but also why they solved the problems the way they did, and what influenced their reasoning.

Materials

In this study, the learners were given a word problem-solving (PS) task, whose solutions depended on realistic factors connected to the problem situation. The PS task was the modelling problem adapted from Verschaffel et al. (2009). The researcher was present throughout the problem-solving process, and learners were encouraged through questioning to verbalise and/or write down the reasoning process they employed to arrive at and justify a particular solution.

Problem Solving (PS) Tasks

PS1: Two boys, Sibusiso and Vukile, are going to help Sonwabo rake leaves on his plot of land. The plot is 1200 square meters. Sibusiso rakes 700 square meters during four hours and Vukile does 500 square meters during two hours. They get 180 rands (R) for their work. How are the boys going to divide the money so that it is fair?

PS2: John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometre?

Procedure

The two problem-solving (PS) tasks were read aloud to all groups of learners by the researcher. All the learners also received the task in written form. The tasks were given to the learners after they were introduced to discussion and argumentation as a strategy to engage in problem solving, and connecting mathematics classroom with the outside world. After attempting the problems individually, learners were then introduced to discussion as a strategy to make sense of word problems, and engage fully in classroom discourse. The group interactions were videotaped and later analysed. The researcher was readily available throughout the session in order to neutrally assist in cases where learners were stuck and to record verbal reasoning from different groups, and learners' justifications of their responses.

RESULTS AND DISCUSSION

The word problems above, which are examples of a central part of mathematics learning, can be seen as attempts to connect mathematical reasoning to everyday life. In other words, the PS task can be viewed as a manifestation of the notion that mathematics is or should be part of mundane practices in everyday life. The results of this study illustrate that students acted in a complex situation when attempting to solve these problems in an ambiguous reality. Learners responded to PS1 and PS2 tasks by using different models and approaches (see Extracts below) that illustrated different interpretations and use of real world experiences in their problem-solving.

Results of PS1

The following extract shows how learners interpreted and solved the first problem (PS1), and describes their reactions after being prompted by the researcher.

Extract 1

Learner 5 (L5): Because they both worked, I will just give them the same amount.

Researcher (R): Any other suggestion? How do you think they should divide the money fairly amongst themselves?

L5: I will still share it equally because there's no need to take more money than her.

R: OK...

L6: To be fair I will give the one who raked 700 meters R100 and the other one R80.

R: What do you think about he who raked for 2 hours?

L1: It is not fair,... one did it in shorter time ..., and the one worked in 4 hours and did 700 square meters, so will first have to calculate the time and then divide up.

In general it was observed that all participants found the problem very difficult to solve. Learners encountered difficulties in making sound and reasonable assumptions concerning what it means to share or divide the money in a fair manner. The discussions are characterised by learners engaging in realistic considerations (Verschaffel et al. 2000), as a result of solving a problem in an ambiguous reality.

From some of these learners' socio-cultural perspectives, dividing the money fairly simply translates to sharing the money equally, as seen in the text of Extract 1, where learner 5 said: "*Because they both worked, I will just give them the same amount*". The fact that this statement could legitimately be challenged by other learners suggests that learners enter mathematics classrooms from a range of socio-cultural backgrounds. Interestingly, dissatisfaction with this response also suggests that there is a specific culture of calculation represented in and through the practices embedded within the mathematics classroom, and learners whose socio-cultural background is congruous with that classroom culture are more likely to be construed as successful learners (Zevenbergen 2000).

Mathematizing as Communicative Work

The data show that learner discussions during the focus group moved back and forth between the problem-solving strategies that they employed and making sense of situations used in problem statements. What appears to be evident from the texts in Extract 1 is that culture and real-life knowledge played a pivotal role in learners' mathematical reasoning and problem-solving in relation to this task. Table 1 shows different models that were used by the learners in different groups, when attempting to solve this task (PS1).

Table 1: Models suggested for sharing money in learners' responses models for sharing

	Models for sharing	*Frequency (n=107)*
A.	Divide equally (R180/2)	45
B.	Amount of work done	32
C.	Time taken to do work	11
D.	Payment by performance	7
E.	Other	12

The data in Table 1 reflect a highly frequent response in which learners propose sharing the money equally. One may legitimately assume that, as noted earlier, the high number of responses suggesting that the best solution is sharing the money equally stems from a specific interpretation (out of multiple meanings) of the word ‘fair’, influenced by the learners’ socio-cultural backgrounds.

Calculations Using Magnitude of Work Done

It is very interesting to see that there were further suggestions, beyond sharing or dividing the money equally, which can be taken as an indication that learners used diverse classroom mathematical experiences and real-knowledge skills acquired through life-experiences. The data show that majority of the boys suggested the alternative model of sharing the money by calculating the amount of work done (without considering the time taken as a factor). Although one cannot make conclusive claims from the data, it is well-known that in African cultures, there are certain jobs that are only reserved for boys and those that are reserved for the girls. Practical experience of the labour involved might explain the higher demand on the part of boys for a different kind of distribution. In short, boys and girls probably used different real-life knowledge and experiences in suggesting a model to solve this problem.

Extract 2

L6: To be fair I will give the one who raked 700 meters R100 and the other one R80.

L7: If we divide the piece of work done by the total ground that was raked, we have 700 divide by 1200, which gives 7/12. Then we multiply 7/12 by R180, the money to be shared, which gives R105. So one should have R105 and the other one gets R75.

The text in Extract 2 shows how two boys solved this problem. Both learners L6 and L7 considered the amount of work as a key factor of sharing the money fairly. It is clear that Learner 6 estimated the proportions of the money to be shared. His estimation is not far from learner 7’s solution statement. Learner 7 used the concept of decimal fractions to solve this problem based on amount of area raked by each boy, and sharing the money according to the fraction equivalent to the work done. It is also evident that language had no effects in learners’ interpretation of this word problem.

Compared with the situation in which Western students (for example, Sweden and United States) have been challenged by the problematic word problems (Greer 1997; Säljö te al. 2009; Verschaffel et al. 2009), the data in Table 1 show that some South African learners are not in the position to, as Freudenthal (1973) puts it, “mathematize” the world by means of elementary forms of mathematical modelling. The data in the Table 1 shows that the majority of the South African learners in this study failed to argue and make counter-arguments beyond equating a “fair sharing” of the money with “dividing money equally”.

Table 2: Learners’ group interactions when solving a PS task

Model			
Amount of work done	*Time*	*Pay by performance*	*Other*
L(earner) 4: They both worked, I’ll just give them same equal amount.	*L1:* …the other one did in shorter time…so we’ll have to calculate the time and then divide up.	*L5:* ...Vukile must have more than R90 because he did it in a very short space of time...	*L3:* Because they are friends, I will share it equally because there is no need to take more money.…
L6: To be fair, I’ll give the one who 700 square meters R100 and the other one R80	*L5:* ...one worked the smallest part in a short time, so the other one used much more time working in a bigger place...		
L2: No...Vukile only did less work...	*L2:* But Sibusiso raked for four hours and Vukile for just two hours, how can you share money equally?	*L5:* I agree, but he did it faster than Sibusiso...	
L4: R180/2 is R90 for seach of them, it’s fair...			

The following extract reflects the recorded arguments made by the ninth grade learners when engaging in solving the PS1 task.

The text in Table 2 is an example of one of the episodes recorded during PS group discussions and interactions. During this activity, learners were encouraged to discuss in the language of their choice, and they used predominantly English to solve the task, with rare occasions of code-switching. Data in this extract illustrate that learners moved between the models and computations without noticing what premise applies in each case. The utterances in Table 2 clearly suggest that learners' reasoning occurred between the proposed models, with each member of the group failing to reason beyond one model. As such, the counterarguments against one model often came from a different model without comprehending that the premises for the reasoning and calculations have changed.

In the fourth and fifth utterances in Table 2, we see how L5 moves between the two models without taking into account that they are different in terms of their propositions and implications to fair sharing of the money between the two boys.

Although all the group members (six learners in this group) participated in dialogue and talk, they could not arrive at a common solution to this problem. Rather, it was evident that the quality of arguments and nature of justifications improved over time, as they continued to engage in mathematical modelling of PS1.

Results of PS2

Word problems are often the only means of providing learners with basic pragmatic or common-sense experience in problem-solving and mathematization (Reusser and Stebker 1997). The PS2 represents one of many questions that are used for assessments in South African mathematics classrooms, and elsewhere in the world (see Verschaffel et al. 2009). All the learners' solutions were classified into three main categories based on their written answers and verbal responses to the interview questions. In fact, the PS2 task has a mathematical structure that is related to real-life factors. In other words, the solution of this problem depends on the rate of progress influenced by factors such as physical strength, preparedness, weather, fatigue, etc.

In solving the PS2 task, learners failed to reflect common-sense understanding of reality in problem-solving. The majority of learners answered "170 seconds" to this problem, consistent with findings of many studies conducted in Europe and Asia, for a wide variety of problems across different linguistic and cultural settings (see for an example Schoenfeld 1991; Verschaffel et al. 1994, 2000, 2009). In this case, learners simply read and converted the text into a mathematical operation in fairly direct manner, without considering more carefully in what manner the text information is to be translated into a mathematical form in order to be successful. In responding to this problem, learners simply multiplied 17 seconds by 10 to find out how long it takes to run a kilometre. This "runner problem" has been used in a number of studies in different parts of the world, and the results, consistent with the results of this study, are thought-provoking: "...the percentage of students in the various countries who gave the unqualified answer 170 seconds ranged from 93% to 100%" (Verschaffel et al. 2000: 44).

The Nature of Justifications

Extract 3 demonstrates learners' responses to PS2 task, and the nature of justifications made by learners before and after being prompted by the follow-up interviews.

Extract 3

L5: I multiplied 17 by 10 it gives me 170, and then I got my answer.

R: Your solution may not work, because of real life factors. Why did you solve the problem that way?

L1: I don't agree with L5, because it's a kilometre, when you run 100 meters you run with your full speed, but then at 17 you cannot run your full speed, you have to a bit sometimes jog because this is a kilometre it's not 100 meters...

R: Okay...

L1: Mathematically it's correct but in real life it's not going to be like that, it's going to be much longer, it's not going to be a 170 seconds.

The learners' responses to the researcher's question in Extract 3 demonstrates that learners could justify their responses in terms of their own interpretations of the problem situation when confronted with, what Inoue (2009) refers to as, the "irrationality of their responses". Extract 4 shows the justification that learner 1 presented after she was prompted to do so in the interview question. Learner 1 suggests that although the solution is mathematically correct, "in real life it's not going to be like that", as she reasons that in a real life situation it will take John "much longer" to run the 1 kilometre distance.

This learner acknowledges the disconnection that exists between what she learnt in classroom mathematics and real-life problems that are not related to the mathematics discourse that she is exposed to. In so doing, her newly acquired argumentation and discussion skills assist her in affirming the mathematical solution offered by learner 5, and in the process linking the mathematics to real knowledge by suggesting that "...when you run 100 meters you run with your full speed, but then in this case you cannot run your full speed, you have to jog a bit sometimes, because this is a kilometre it's not 100 meters... This reasoning is largely influenced by consideration of realistic factors that exist in real life situations. As can be seen in Table 3, there were different justifications of seemingly 'unrealistic' responses. These responses reflect a sample of justifications that were presented by the learners spontaneously (in response to the second interview question) as well as examples of justifications after being prompted explicitly.

Table 3: Learners' sample justifications of 'unrealistic' responses

Spontaneous justifications	*Justifications after being prompted explicitly*
John is a well trained runner. He can make this time.	I don't think that is possible, because if run too much you will get tired and your speed will decrease, so as the speed slows down the time goes bigger.
If he is fit as expected, John can maintain his best 100m time in a kilometre distance.	When you run 100 meters you run with your full speed, but then at 17 you cannot run your full speed, you have to a bit sometimes jog because this is a kilometre it's not 100 meters

It's not true because in the first 100 meters you running your full speed, but when the time goes on you get tired

If John is a super-fit athlete, who trains regularly with a coach, then he can make it on time

Similar to studies conducted by Inoue (2009) on an introductory-level psychology class in Southern California, most of the learners' justifications were based on the claim that common-sense real life factors do not necessarily apply to certain or particular mathematical and/or classroom situations. In so doing, justification of computational answers were designed to make their responses reasonable and acceptable.

The data showed that these learners are used to this kind of problem, particularly in natural science studies. Moreover, the text in extract 3 shows that learners are exposed to classroom settings where simply providing the "correct" answer to a structured problem is sufficient. This was noticed when learner 5 answered: "I multiplied 17 by 10 it gives me 170, and then I got my answer", without providing justifications and checking whether the answer is reasonable. A prominent finding in most of the research of this kind is that learners' performance on word problems differs dramatically depending on how the problems are designed (see Verschaffel et al. 2000). The PS2 task is formulated according to the standard expectations in mathematics teaching, and, within this discourse, it can be solved correctly through a straightforward operation such as division or multiplication.

It is fundamentally necessary to draw reality into mathematics classrooms by starting from learners' everyday-life experiences and situations, if one aims to teach learners to connect classroom mathematics to real-life knowledge in their thinking and reasoning. The inclusion of application and modelling problems is intended to convince learners to develop the necessary skills of knowing when and how to apply their classroom mathematics effectively in situations encountered in everyday life. The researcher contends that this goal can only be realised if learners and teachers bring reality into mathematics (that is, view everyday life situations and learners' experiential reality as a natural extension of teaching and learning formal school mathematics) and conversely bring mathematics into reality. The researcher believe that engaging learners in mathematical reasoning, using available and relevant real-world contexts that are familiar to them and/or related to their own daily experiences permits them to deepen and broaden their understanding of the usefulness of mathematics, and may influence sound mathematical conclusions that make sense in out-of-classroom contexts. In other words, learners should be regularly encouraged to identify an immense variety of situations as mathematical situations, in the process of learning a variety of ways of thinking mathematically.

Exposing learners to word problems that are familiar to them, like the problems used in this study, may be viewed as an attempt to establish a new classroom culture through new socio-mathematical norms. Such problems provide learners with the opportunity to model and 'mathematise' a problem situation, and not primarily to apply a ready-made solution procedure without realistic considerations. This is not at all to imply that knowledge of solution procedures is not relevant, it serves only to stress that the primary objective is to make sense of the problem. As noted earlier in this paper, learners were encouraged to use and justify their own sense-making methods, exploring the usefulness and soundness of their suggested models with regard to the problem. In the process of presenting arguments and counterarguments, learners are stimulated to articulate and reflect on their cultural or personal beliefs, alternative conceptions and effective strategies to solve problems.

CONCLUSION

The pedagogical conclusion of this study is that cognitive power produced by multilingual mathematics classrooms settings has a strong influence on learners' realistic problem solving, and implicit beliefs and rules relating specifically to learners' mathematical activities, hinder learners from using realistic knowledge in their solutions. Finally, this study illustrated that looking closely into learners' justifications of their ostensibly unrealistic solutions can inform us of the various ways in which they elucidate and make sense of the problem situation as well as the nature of problem solving activity.

RECOMMENDATIONS

In this paper, the researcher has discussed an example (word problem-solving) of the kind of classroom activity that connects classroom mathematics to everyday-life experiences and knowledge of the learners. The researcher views the introduction of new socio-mathematical norms in mathematics classroom as an attempt to create a substantially reflective teaching and learning environment. Teachers should be aware of the fact that the context of mathematics schooling and the real world context are fundamentally different. What this study tried to illustrate is that mathematics word problems become more complex when the relationship between the mathematical operations and the verbal formulations are not of the standard kind.

Mathematics classrooms in South Africa consist of learners from different cultural and social classes. Individuals in these classrooms are engaged in different kinds of discourses that sometimes overlap and at times are mutually exclusive. Consequently, classroom stakeholders should be in a position to choose discursive practices that promotes mathematical problem-solving and arguing in a particular setting.

Overlaps and/or moves between discourses in the mathematics classroom is sometimes complicated, as illustrated in this study in relation to the concept of 'fair sharing of money', discussed earlier in this paper. Teachers should be in a position to assess what the logic of the argumentation is and what are useful arguments at a particular point in time.

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